RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. FIRST SEMESTER EXAMINATION, DECEMBER 2013

FIRST YEAR

Date : 17/12/2013

Math for Eco (General)

Time : 11am – 2pm

Paper : I

Full Marks : 75

(Use a separate answer book for each group)

Group – A

Answer **any eight** questions of the following : (8 × 5)

1. X be a non-empty set. Define a relation on power set of X i.e. on P(X) by A ρ B iff "A is a subset of B", A, $B \in P(X)$ check whether the above relation is an equivalence relation? (5)
2. i) Define injective and surjective mapping from a set X to another set Y. (3)

   ii) $f(x) = x^2 - 1$, $x \in \mathbb{Z}$ where $\mathbb{Z}$ is the set of integers. Is f(x) injective? (2)
3. Show that a mapping $f : X \to Y$ is invertible iff it is a bijective mapping. (5)
4. Given three sets A, B, C describe the set from of the following :

   i) The set contains elements from only one the sets.

   ii) The set contains elements from at least two of the sets.

   iii) The set contains elements from at least one of the sets.

   iv) The set contains elements from not more than three sets. (5)
5. i) Define supremum (least upper bound) and infimum (greatest lower bound) for a non-empty set X. (2)

   ii) Find supremum and infimum of the following sets. (3)

   a) The set of all natural numbers.

   b) $\{x \in \mathbb{R} \mid x^2 < 1\}$.
6. i) State Archimedean property and Density property of real numbers. (2)

   ii) Assuming that between any two real number there exist a rational number, prove that between any two real number there also exist an irrational number. (3)
7. If $a, b \in \mathbb{R}$ and $0 \le a - b < \in$ holds for every positive $\in$, prove that $a = b$. (5)
8. Prove that union of open sets is open set. (5)
9. Is the union of any collection (finite or infinite) of closed set is a closed set? (5)
10. Find the set of limit points of the following subsets of $\mathbb{R}$.

    a) The set of natural numbers. (1)

    b) $\{m + \frac{1}{n}, m \in \mathbb{N}, n \in \mathbb{N}\}$, where $\mathbb{N}$ is the set of natural numbers. (2)

    c) $\{x \in \mathbb{R} \mid x^2 < 1\}$. (2)
11. Consider the function,

    $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^+ \cup \{0\}$ defined by $d(x, y) = |x^2 - y^2|$,

    Is d a metric function on $\mathbb{R} \times \mathbb{R}$. (5)
12. Prove that a monotone increasing sequence of $\mathbb{R}$ which is bounded above is convergent. (5)
13. Prove that every convergent sequence is a cauchy's sequence. (5)
14. Test the convergence of the following series.

    $\frac{1}{1.3} + \frac{2}{3.5} + \frac{3}{5.7} + \ldots\ldots\ldots\ldots\ldots\ldots$ (5)

Group – B

Answer **any seven** questions of the following: (7 × 5)

15. State De Moivre's theorem. Use it to solve the equation $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$. (1 + 4)

16. Prove that $\sin\left[i\log\frac{a-ib}{a+ib}\right] = \frac{2ab}{a^2+b^2}$. (5)

17. Prove that a real (or complex) square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix. (5)

18. Show that the matrix $\begin{pmatrix} a+ib & c+id \\ -c+id & a-ib \end{pmatrix}$ is unitary if $a^2+b^2+c^2+d^2=1$, where a, b, c, d $\in \mathbb{R}$ (5)

19. State the Jacobi's Theorem. Use it to prove that $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$ (5)

20. If ω be a cube root of unity, prove that $(a+b\omega+c\omega^2)$ is a factor of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ (5)

21. Reducing the given matrix to it's normal form, find it's rank :

$$\begin{pmatrix} 3 & -2 & 0 \\ 0 & 2 & 2 \\ 1 & -2 & -3 \end{pmatrix}$$ (5)

22. Prove that the intersection of two subspaces of a vector space is also a subspace. (5)

23. Define the basis of a finite dimensional vector space. Prove that the vectors (1, 2, 1), (2, 1, 0) and (1, -1, 2) form a basis of the vector space $\mathbb{R}^3$ over $\mathbb{R}$. (1 + 4)

24. Find a basis of $\mathbb{R}^3$ containing the vectors (1, 1, 2) and (3, 5, 2). (5)

25. Find the co-ordinate vector of $\alpha = (1,3,1)$ relative to the basis (1, 1, 1), (1, 1, 0) and (1, 0, 0) of $\mathbb{R}^3$. (5)